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Tradable Permits Schemes and New Technology Adoption

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Tradable Permits Schemes and New Technology Adoption*

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Abstract

In this paper technology adoption behaviour under (regulatory) no–anticipation of new technology, and imperfect competition in a tradable permits scheme (rents market) is investigated. The inter–dependence between the incentive to adopt a new technology and the allowance price is explicitly modelled. A firm's long–term incentives to adopt a new technology depend on the future value of tradable permits (scarcity rents) and, ultimately, on the level of uncovered pollution emissions (permits demand) and the level of offered emission permits (permits supply) – both affected by the current technological status. In an imperfectly competitive permit market, the aggregate supply is the solution of a non–cooperative game that possesses a pure-strategy Nash equilibrium. It is shown that this condition is also satisfied when a price–support instrument, which is contingent on the adoption of the new technology, is introduced. This is done to foster the firms' long–term incentives to adopt new technologies. The impact of the price–support contract on the scarcity rent value and on the technology adoption behaviour is both theoretically and numerically examined: (i) it creates a floating price floor that can be interpreted as a minimum price guarantee; (ii) the higher the minimum price guarantee, the higher the aggregate level of adoption and the earlier the adoption of new technologies.

JEL classification: D8, H2, L5, Q5.

Keywords: Contingent policy; Emission permits; Floating Price Floor; Long-term incentive; Price Support Contract.

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1 Introduction

Under no–anticipation of new technologies, the original policy levels of a tradeable permits regime are in general no longer optimal as soon as a firm adopts a new technology. It has been shown that this discourages future adoption of new technologies and suggests the consideration of ratcheting policy stringency over time. Policy levels, however, tend to remain constant for long periods of time making the long–term incentives to adopt new technologies under tradable permits an interesting question to explore. We first characterize the inter–dependence between the incentive to adopt a new technology and the future permit price, one of the relevant driving factors of new technology adoption. Then, we investigate the technology adoption strategies and the permit–trading strategies of regulated firms when policy levels remain constant for long periods of time and the assumption of perfect competition in the permit market is relaxed. Finally, we introduce a mechanism that fosters the long–term incentives to adopt new technologies under no–anticipation of new technology

It has been shown in the literature that tradeable pollution permits may not be the most desirable policy instrument in a dynamic setting, even if there is complete information and the permit market is perfectly competitive – see Milliman and Prince (1989a), Malueg (1989), and Biglaiser et al. (1995). The reason being that the original policy levels may astray from optimality if new technologies are adopted. In principle, under a tradable–permits regime a regulatory agency chooses a fixed, total amount of permits so that the marginal private cost of pollution control is equal to the pollution's social damage. Polluters typically respond to such regulation by investing in pollution–reducing capital. Suppose now a new, advanced technology becomes available. As investments in the new technology take place, the costs of pollution control fall, reducing the value of the permits (scarcity rents). The system deviates from what the regulatory agency initially deemed optimal. By adjusting the total number of permits, however, the marginal private cost of pollution control and the pollution's social damage may be equalized again. Yet, in reality an amendment of a policy level is the exception rather than the rule. The arrival of a new technology is hardly predictable and, typically, once the regulatory agency implements its policy, levels remain constant for long periods of time. The European Union Emission Trading Scheme (EU ETS), EU's flagship market–based policy for capping carbon emissions, constitutes a blatant example of a firm (unresponsive) policy level. Currently, permit prices reflect an extremely low demand for permits in the market. There has been considerable attention on what needs to be done in order to support long–term investment incentives and facilitate the orderly functioning of the system. Several options have been listed.[1] To reduce the immediate surplus, the European Commission proposed to temporarily take allowances out of the trading system. This "backloading" is a temporary, potentially insufficient measure. It could (politically) pave the way for structural measures which would tighten the demand–supply balance for emission allowances. However, there are strong reservations about changing the policy by tightening the cap, making the political discussion about amending the policy levels an extraordinary long and impervious legislative process.

While our primary motivation lies with the adoption of new technology, our ideas carry over to any form of interplay between a pure quantity–based scheme and complementary policies that involves the regulation of the same underlying pollutant. Lately the European Commission has been grappling with the perverse consequences of the interaction between the EU ETS and EU energy policies, where the impact of energy efficiency and renewable energy incentives was to lower demand for permits with an obvious impact on permit prices. The interplay between ETS and (non–anticipated) complementary policies constitutes, therefore, another interesting application of the framework proposed in this paper.

In Malueg (1989), Milliman and Prince (1989a) and Jung et al. (1996), tradable–permits regimes analysis has been centered on the aggregate cost savings resulting from an industry–wide adoption of some new technology, where the authors assume that all the firms adopt the new technology, and then they compare aggregate costs before and after technology adoption. More recently, Requate and Unold (2003)

[1] For an overview of the proposed market reform we refer to the report by the European Commission Commission (2012) and to the responses to the Commission consultation by Marcu (2013) and Taschini (2013).

and Requate (2005a) have shown that these aggregate cost savings differ substantially from a single firm's incentives to adopt a new technology. It is observed in these papers that decreasing permit prices (scarcity rents) provide incentives for some firms to free–ride on the other firms' technology investments by taking advantage of decreasing compliance costs via permit purchase. Hence, in equilibrium both types of firms, using old and new technologies, are active in the market. Focusing on the firms' incentives to adopt a new technology, this recent literature attempts to determine how many firms will invest if new technologies becomes available and the policy levels remain constant for long periods of time. The number of firms investing in new technologies is determined endogenously in a framework where firms behave as price–takers on the permit market and are sufficiently small, relative to the whole market, that they cannot affect the market price for permits by adopting a new, less polluting technology. We modify such an assumptionand investigate the long–term investments' incentives. We assume that the size of the firms' productive projects is fixed and concentrate on the effects that fixed policy levels have on the adoption of new technologies. Hence, pollution emissions are constant, but we allow for uncertain cash–flows. Firms have two mutually non–exclusive compliance options: adoption of a new technology, which we assume is an irreversible investment, and exchange of permits. When firms cannot fully offset their emissions by trading permits, they face a fixed, per–unit penalty for excess emissions. Extending Requate and Unold (2003),firms in our model view their investment decisions as affecting the market price of permits. Hence, the incentive to adopt a new technology hinges on the future permits' scarcity. This depends on the aggregate demand and the aggregate supply of allowances and, ultimately, on the current technological vector, i.e. the technological state under which each of the firms operates. This allows us to close the loop and explicitly model inter–dependence between technology adoption and the permit price: the number of firms that decide to adopt the new technology depends on the (rational) expectations of the future permit price; and the future price of permits is contingent on the number of firms that adopt the new technology. When the firms' choices can affect the market price, we argue that there is a natural trade–off between offering a higher number of cheap permits, or less of them, but at a higher value[2]. Consequently, we show that the aggregate supply is not necessarily equal to the number of unused permits. Instead, the aggregate permit supply is the solution of a non–cooperative trading (Nash) game.

Under no–anticipation of new technologies, Requate (2005a) and references therein show that the original optimal policy levels are no longer so when firms adopt new technologies: As investments in new technologies take place, the costs of pollution control fall. This leads to a depreciation of the permit price because the policy level (the aggregate amount of permits) does not adjust to the modified conditions. Altering the allocation of permits ex–post may be the simplest route to circumvent this issue. An amendment of a policy level, however, is a rather exceptional event. When policy levels remain constant for long periods of time, an internal mechanism that responds to the adoption of new technologies could be a viable option. One such promising mechanism (suggested by Roberts and Spence (1976), Laffont and Tirole (1996), and Biglaiser et al. (1995)) consists of offering guarantees of the future allowance price. This could be organized in the form of a regulator who stands ready to buy permits in the market at an announced price. We consider a similar mechanism: a contract that is written on the final holdings of permits and is contingent on the technology status. Hence, by adopting new technologies, the firms virtually create put–option like contracts, which we dub *Cash–for–permits* and denote by *C4P*. Once created, these contracts are assumed to be non–transferable. We show that this mechanism creates a floating price floor. Moreover, the Nash–games methodology that we use to model the generation of the allowance prices in the no–anticipation setting can be extended to this one. This allows us to show, theoretically and numerically, that the C4P generate a price floor that can be interpreted as a (floating) minimum price guarantee. The comparison of the evolution of the technological vector under different policy setups, however, is not straightforward. We numerically examine the impact of the minimum price guarantee on prices, as well as on the number of firms that adopt low pollution–emitting technologies, together with the adoptions' timing.

[2]This bears some similarities to the problem of large portfolio liquidation

2 The Model

In this section we present our model of a pollution–constrained economy, which consists of a group of polluting firms $\mathcal{I} = \{1, \ldots, m\}$ that operate under a transferable–permits system. We consider a discrete–time, finite–horizon setting, where the interval $[t, t+1]$ denotes one regulated period and $[0, T]$ denotes the regulated *phase*. We assume an exogenous policy set forth by a regulator. Our main aim is to examine the incentives to adopt *new*, low pollution–emitting production *technologies* when the policy levels are constant for a long period of time. The entire allowance schedule, $\{N(t)\}_{t=0}^{T-1}$, is set at time $t = 0$, and $N_i(t)$ denotes the number of permits issued to firm i for use in period $[t, t+1]$.[3]

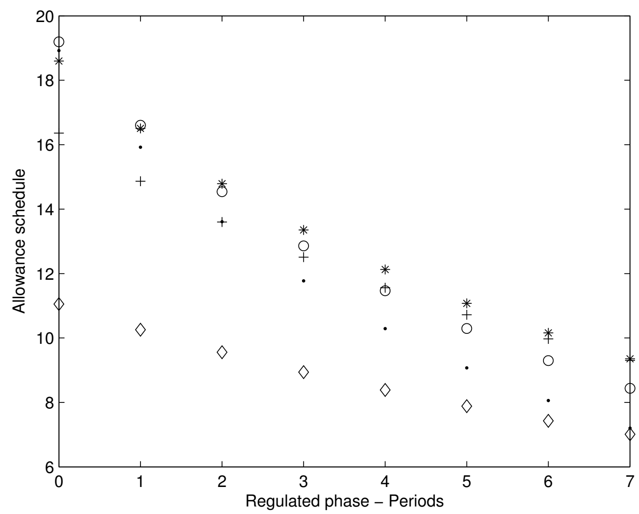


Figure 1: Firm-wise allocation of permits with seven periods and five regulated firms.

Firms have two mutually non–exclusive compliance options: adoption of new technologies, which we assume may occur at most once per firm during the phase,[4] and exchange of permits. When firms cannot fully offset their emissions by trading permits, they face a fixed, per–unit penalty, P, for excess emissions.[5] At the beginning of each regulated period, firms that have not adopted a new technology must decide whether or not they adopt it. The corresponding investment cost, which we denote by C_i, is irreversible and it comes into effect instantaneously. The costs associated to the adoption of a new technology and the exchange of permits are compliance costs; whereas the costs associated to the penalty payment are non–compliance costs. Hence, it is possible to model compliance and non–compliance strategies. In particular, when non–compliance costs are smaller than compliance costs, firms would be better off paying the total penalty for uncovered future emissions, rather than investing in new technology or buying permits. Conversely, when non–compliance costs are high, the firms' incentives to adopt new technologies are determined by their (rational) expectations of the future values of the permits.

Given that we wish to focus our attention on the effects that the policy has on the adoption of new technologies, we assume for simplicity that the sizes of the firms' productive projects are fixed. We do,

[3]The total amount of tradable permits and its allocation schedule are selected by equating the marginal private cost of pollution control and the pollution's social damage. As shown by Biglaiser et al. (1995), among others, the optimal allowance schedule should be decreasing over time, as graphically illustrated in Figure 1.

[4]The adoption of the new technologies generates a permanent reduction in emissions. We assume a lump–sum investment cost. The case of a costly choice of different levels of permanent reduction is left for future research.

[5]As discussed in Jacoby and Ellerman (2004), in such a framework the penalty payment constitutes an alternative to compliance.

however, allow for uncertain output cash–flows. The variability of the latter could obey, for instance, unanticipated demand shocks that affect output prices. The adoption of a new technology is assumed to affect only the amount of emissions, but not production. We model the idiosyncratic shocks to the firms' profits from production via the independent random variables R^i that have the following distribution:

$$R_i = \begin{cases} R_i^u, & \text{with probability } q_i, \\ R_i^d, & \text{with probability } 1 - q_i, \end{cases}$$

where the q_i's are given and $R_i^u > R_i^d > 0$. The cash–flows of firm i over each period are given by R_i.[6]

In the sequel we use the notation $h_i \in \{o, n\}$ to express whether a firm operates under an old or new technology, respectively. The *technology vector* $h := (h_1, \ldots, h_m)$ indicates the technology regime under which each firm operates.[7] The (cumulative) emissions Q_i of firm $\{i, i \in \mathcal{I}\}$ at date $t+1$, conditional on the firm operating under technology h_i in period $[t, t+1]$ are:

$$Q_i(t+1, h_i) = p(h_i) \cdot Q_i(t),$$

where the *pollution factor* p satisfies $1 < p(n) < p(o)$, and $Q_i(0)$ is given. The decision regarding technology adoption takes place at the beginning of each period. We define the *adoption times*:

$$\tau_i := \min \big\{ t \in \{0, \ldots, T-1\} \mid p(h_i) = p(n) \big\}.$$

Emissions are verified at the end of each period, when permits are also traded. The difference between emissions and allocated allowances, $x_i(t+1, h_i) := Q_i(t)(p(h_i) - 1) - N_i(t)$, determines firm i's net position in the permits market at time $t+1$, for a given technology h_i. If this quantity is positive, firm i should purchase the required permits in the market or pay a per–unit penalty. If this difference is negative, firm i can sell the unused permits for a profit. Let

$$s(t+1, h) := \Big\{ i \in I \,\Big|\, x_i(t+1, h_i) < 0 \Big\}, \quad \text{and} \quad d(t+1, h) := \Big\{ i \in I \,\Big|\, x_i(t+1, h_i) \geq 0 \Big\},$$

be the supply and demand sides of the market, respectively. In the related literature, firms adopting (not adopting) new technologies are usually automatically assigned to the supply (demand) side – see Requate and Unold (2003), among others. In contrast, here the type of firms depends on the permits allocation and the technology vector h. Using the supply and demand definitions above, we introduce the expressions for the number of unused permits and the number of non–offset emissions:

$$\mathcal{S}(t+1, h) := -\sum_{i \in s(t+1, h)} x_i(t+1, h_i) \quad \text{and} \quad \mathcal{D}(t+1, h) := \sum_{i \in d(t+1, h_i)} x_i(t+1, h).$$

The incentive to adopt a new technology depends on the future price of allowances (scarcity rents). In particular, the payoff stream generated by permits sales, permits purchase, and penalty payments, depends on the future permits scarcity. The latter hinges upon the aggregate demand and the aggregate supply and, ultimately, on the current technological vector. To better understand how the inter–dependence between technology adoption (technological vector) and permit value affects the long–term incentive to adopt a new technology, let us describe the different possible firms' permit positions: A low permit price makes the adoption of new technologies a non–viable compliance strategy. Firms in permit need would find compliance by means of allowance purchase a cheaper alternative. Conversely, a high permit price increases the potential profits from unused permits sales and makes the adoption of new technologies a more attractive option. However, the greater the number of firms that adopt a new technology, the lower the permits demand (or the higher the permits supply) and, consequently, the lower the permits value. Hence, the permits value (scarcity rents) depend on the level of uncovered pollution (permits demand),

[6]The case where R_i is a (time–dependent) stochastic process can be incorporated without further difficulties.
[7]Notice that the technology vector represents a possible combination of the firms' technology adoption strategies.

and the level of unused allowances present in the exchange market (permits supply). We argue below that the level $\mathcal{S}_e(t+1,h)$ of permits that is submitted to the exchange is in general smaller than $\mathcal{S}(t+1,h)$. The exchange value of an allowance is determined by the supply–demand ratio:

$$\mathcal{R}(t+1,h) := \begin{cases} -\frac{\mathcal{S}_e(t+1,h)}{\mathcal{D}(t+1,h)}, & \text{if } \mathcal{D}(t+1,h) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

To account for a lower sensitivity of the allowance value in case of extreme permit demand, i.e. the ratio is close to 0, or extreme permit supply, i.e. the ratio is close to 1, we define for $a > 0$ the parameterized family of (reaction) functions $\eta_a : [0,a] \to [0,1]$ as:

$$\eta_a(x) := \begin{cases} \exp\left\{\frac{x^2}{x^2-a^2}\right\}, & \text{if } x \in [0,a), \\ 0, & \text{otherwise.} \end{cases}$$

Figure 2 shows the graph of the function $\eta_a(x)$ for $a = 1$.[8]

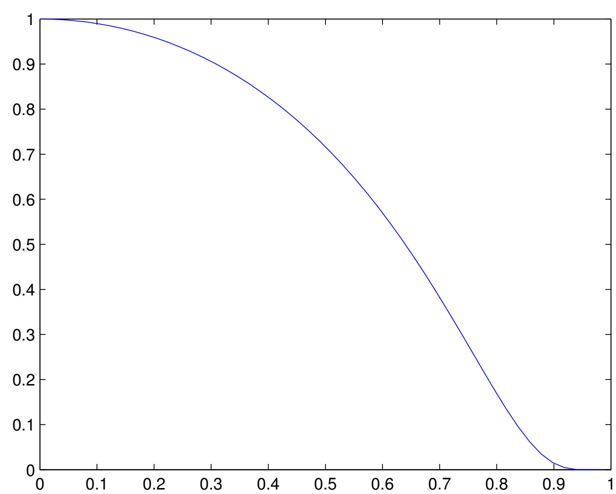


Figure 2: The plot of η_1.

Analytically, the exchange value of the allowance is a function of the compliance costs (supply and demand) and non-compliance costs (penalty):

$$\begin{aligned} \Pi(t+1,h) &:= P \cdot \eta_{\mathcal{R}(t+1,h)}\Big(-\frac{\mathcal{S}_e(t+1,h)}{\mathcal{D}(t+1,h)}\Big) \\ &= P \cdot \exp\Big\{\frac{\mathcal{S}_e(t+1,h)^2}{\mathcal{S}_e(t+1,h)^2 - \mathcal{D}(t+1,h)^2}\Big\}. \end{aligned} \qquad (1)$$

One should observe that the indifference buy–price for an allowance that safeguards a firm that is in permits shortage from paying the penalty P is precisely P. By construction the exchange value of an allowance satisfies $0 \leq \Pi(t+1,h) \leq P$.

[8]The functions η_a are the right halves of scaled mollifiers (see Evans (1998)). They are infinitely smooth at 0 and a, with derivatives of all orders at these points equal to zero.

3 Trading strategy and no–anticipative technology adoption

In a tradable permit scheme where firms can adopt a new technology, the future price of permits depends on the number of firms that adopt new technologies. At the same time, the number of firms that decide to adopt new technologies depends on the (rational) expectations of the future permit price. Most models assume that firms do not view their decisions on technology adoption as affecting the price of permits (see Requate and Unold (2003) and references therein). We relax such an assumption and propose a model where the inter–dependence between technology adoption and permit value is explicitly modelled. Below we describe the mechanisms that govern the firms' decision to trade permits and firms' decision to adopt new technologies.

3.1 Determining the aggregate supply $\mathcal{S}^e(t+1,h)$ and the permit price $\Pi(t+1,h)$

When firms cannot fully offset pollution emissions by trading permits, they face a fixed per–unit penalty, P, for excess emissions. Hence, it is in the buyers' best interest to cover all their non–offset emissions by purchasing permits at any price lower than the penalty P. Consequently, the aggregate demand equals the number of non–offset emissions. On the contrary, it is in the sellers' best interest to control the number of permits offered in the market: the lower the aggregate supply, the higher the exchange value (scarcity rent). There is a natural trade–off between offering a higher number of cheap permits, or less of them, but at a higher value. Consequently, the aggregate supply is not necessarily equal to the number of unused permits. As shown below, the aggregate supply corresponds to the solution of a non-cooperative game.

In order to analyze how sellers choose their supply schedules, we should consider the income generated from the permit exchange of firm $i \in s(t+1,h)$. Let us express this as a function of the *supply vector* $\big(x_i^e(t+1,h), x_{-i}^e(t+1,h)\big)$, i.e. the number of allowances firm i–th would submit to the exchange, and those that would be submitted by the other firms on the sell–side. Namely,

$$\Psi_i\big(x_i^e, x_{-i}^e\big) := x_i^e P \cdot \eta_{\mathcal{R}}\Big(-\frac{\mathcal{S}_{-i}^e + x_i^e}{\mathcal{D}}\Big), \tag{2}$$

where $\mathcal{S}_{-i}^e = \sum_{j\in s\setminus\{i\}} x_j^e$, and we have omitted the arguments $(t+1,h)$ to keep the notation as uncluttered as possible. Equation (2) showcases the trade–off described above: sellers choose between offering a high number of cheap permits, or less of them, but at a higher value. Below, we follow the well–worn trail of studying the best–response correspondences of each seller's response to the remaining sellers' submission of allowances to the exchange. We show the existence of Nash equilibria of this strategic interaction. To this end we start with the following:

Lemma 3.1 *For any supply vector $\big(x_i^e, x_{-i}^e\big)$, the mapping $\tilde{x}_i^e \mapsto \Psi_i\big(\tilde{x}_i^e, x_{-i}^e\big)$ is maximized at a single point. In other words, the correspondence*

$$\Phi_i(x_i^e, x_{-i}^e) = argmax\Big\{\Psi_i\big(\tilde{x}_i^e, x_{-i}^e\big) \,\big|\, \tilde{x}_i^e \in [0, x_i]\Big\}$$

is single valued.

Notice that out of the three requirements to apply Kakutani's Fixed–point Theorem (see for example Meyerson (1991)), Lemma 3.1 takes care of the non–vacuity and the convexity. We then need an upper–semicontinuity result, which we present in the following.

Lemma 3.2 *Let the mapping $\Phi : \mathbb{R}^{m_s} \to \mathbb{R}^{m_s}$ be defined via*

$$\Phi(x_{i_1}, \ldots, x_{i_{m_s}}) := \bigotimes \Phi_i(x_{i_j}, x_{-i_j})$$

for $(x_{i_1}, \ldots, x_{i_{m_s}}) \in \bigotimes [0, x_{i_j}]$, then Φ is continuous.

Lemmas 3.1 and 3.2, together with Kakutani's Fixed–point Theorem imply that the mapping

$$(x_1, \dots, x_{m_s}) \mapsto \Phi(x_1, \dots, x_{m_s})$$

has a fixed point. In other words, we have proved the following:

Theorem 3.3 *The (non–cooperative) game* $\mathcal{G} = \big\{[0, x_i(h)], \Psi_i\big\}_{i \in s}$ *possesses a pure–strategy Nash Equilibrium.*

Moreover, the Nash equilibrium mentioned in Theorem 3.3 is unique. This follows from the fact that it coincides with the unique solution of the system of equations:

$$D_{y_{i_j}} \Psi(y_{i_j}, y_{-i_j}) + \lambda_{i_j} = 0, \quad \lambda^{i_j}(y_{i_j} - x_{i_j}) = 0, \quad \sum_j x_{i_j} \leq 1, \quad j = 1, \dots, m_s,$$

where the λ_{i_j}'s are the Lagrange multipliers associated to the constraints $y_{i_j} - x_{i_j} \leq 0$. From now on $x^*_{i_j}(t+1, h)$ will represent the j–th entry of the Nash–equilibrium that results from the solution of the game among the firms in permit excess, contingent on the technology vector h. The *equilibrium exchange value* of an allowance, contingent on the technology vector h, is:

$$\Pi^*(t+1, h) := P \cdot \eta_{\mathcal{R}(t+1,h)}\Big(- \frac{\mathcal{S}^*(t+1, h)}{\mathcal{D}(t+1, h)} \Big).$$

A further comment should be made regarding the generation of prices in the case where some of the constraints $y^{i_j} - x^{i_j} \leq 0$ are binding. If $x_{i_j} = x^*_{i_j}$, then the following point is of course moot. Otherwise, if some firms are not able to increase their supply schedules up to the (unconstrained) equilibrium level, then the firms that still have availability of permits have extra room to increase their exchange offers. Interestingly, the additional potential number of permits does not restore the original aggregate supply of the unconstrained problem, i.e. the aggregate supply of permits (in equilibrium) in the presence of binding constraints is bounded above by that of the unconstrained problem. When some firms' supply–constraints bind, therefore, the equilibrium price increases. Moreover, the firms with non–satiated constraints collect higher profits than in the unconstrained case by virtue of a lower (aggregate) supply. We formalize these claims in the following:

Lemma 3.4 *Let* $\{x^*_{i_j}(t+1, h)\}$ *be the equilibrium supply profile of the game* $\mathcal{G} = \big\{[0, 1], \Psi_i\big\}_{i \in s}$*, then the equilibrium supply profile* $\{\tilde{x}_{i_j}(t+1, h)\}$ *of the constrained game* $\tilde{\mathcal{G}} = \big\{[0, x_i(h)], \Psi_i\big\}_{i \in s}$ *(with* $x_i(h) < 1$*), and the corresponding price* $\tilde{\Pi}(t+1, h)$ *satisfy:*

1. *If* $x_i(h) < x^*_i(t+1, h)$*, then* $\tilde{x}_i(t+1, h) = x_i(h)$*.*
2. *If* $x_i(h) > x^*_i(t+1, h)$*, then* $\tilde{x}_i(t+1, h) > x^*_i(t+1, h)$*.*
3. $\tilde{\Pi}(t+1, h) > \Pi^*(t+1, h)$*.*

In order to describe how the exchange of allowances is executed in the permits market, we need to specify how buy– and sell–orders are matched: All orders are submitted to a centralized exchange market, in which they are randomly (and uniformly) matched one–by–one. All the sellers' orders are executed. Concerning the demand side, the probability that the orders of firm $i \in d(t+1, h)$ are matched is:

$$-\frac{x_i(t+1, h)}{\mathcal{D}(t+1, h)}.$$

In other words, firm i's access to the sell–side of the market corresponds to its relative contribution to the aggregate demand schedule. Hence, the executed orders of firm $i \in d(t+1, h)$ are:

$$X_i(t+1, h) = \frac{x_i(t+1, h)}{\mathcal{D}(t+1, h)} \cdot \mathcal{S}_e(t+1, h).$$

3.2 The firms' expected payoffs over $[t, t+1]$ from permit exchange

The firms' expected payoffs are computed assuming firms' orders will be matched as described in the previous Section. Let $\phi^i(t+1,h)$ denote the payoff for firm i, contingent on the technology vector h. If $i \in s(t+1,h)$, then firm i would sell $|x_i^*(t+1,h)|$ allowances, and its profit would be:

$$\phi_i(t+1,h) = \Pi^*(t+1,h) \cdot |x_i^*(t+1,h)| + \Delta R_i(t+1).$$

If $i \in d(t+1,h)$, then

$$X_i(t+1,h) = \frac{x_i(t+1,h)}{\mathcal{D}(t+1,h)} \cdot \mathcal{S}^*(t+1,h),$$

and the firm's expected profit would be

$$\phi_i(t+1,h) = \mathbb{E}[R_i(t+1)] - P \cdot x_i(t+1,h)\frac{\mathcal{D}(t+1,h) - \mathcal{S}^*(t+1,h)}{\mathcal{D}(t+1,h)} - \Pi^*(t+1,h) \cdot X_i(t+1,h).$$

The second component, the quantity $x_i(t+1,h)\big(\mathcal{D}(t+1,h) - \mathcal{S}(t+1,h)\big)/\mathcal{D}(t+1,h)$, represents the number of emissions that are not offset using allowances, and for which the the prescribed penalty would be levied, i.e. it corresponds to the non–compliance cost. The third component represents the compliance cost via allowance purchase.

3.3 The firms' expected payoffs over $[t_0, T]$.

At the start of each period, the mechanism that governs the firms' decisions to adopt new technologies is based on all possible evolution paths of the technological vector. If firm i has not adopted a new technology at the start of period t_0, its decision regarding adoption is made by comparing its expected continuation payoffs, which depend on the other firms' (past and future) technological choices, corresponding to adoption and non–adoption. To make this precise, let us define

$$\mathcal{O}(t_0) := \big\{ i \in \mathcal{I} \mid p(h_i) = p(o) \big\}.$$

$\mathcal{O}(t_0)$ is the set of firms that, before the start of period t_0, have yet to invest in a new technology. When assessing whether or not to adopt, firm $i \in \mathcal{O}(t_0)$ must perform a cost–benefit analysis. The direct cost associated to investment in a new technology is clear, but the reduced costs of compliance, and/or the profits arising from trading freed–up permits, depend on the evolution of the other firms' technological status. The latter is uncertain for all firms in $\mathcal{O}(t_0)$: these firms could adopt at any time between t_0 and T, or not at all. In order to model all these scenarios, we consider the following families of row–stochastic matrices of dimension $m \times (T+1-t_0)$:

$$\mathcal{M}(t_0) := \Big\{ M \in \mathbb{R}^{m \times (T+1-t_0)} \,\Big|\, M(j,1) = 1 \,\forall\, j \notin \mathcal{O}(t_0) \Big\}.$$

We shall call $\mathcal{M}(t_0)$ the set of *path matrices*, and each of its elements denotes a possible way in which technology adoption may be undertaken by the $\#\mathcal{O}(t_0)$ firms that can still make such decision at date t_0. The fact that $j \notin \mathcal{O}(t_0)$ implies that $M(j,1) = 1$ for all $M \in \mathcal{M}(t_0)$ should be read as "at the beginning of period t_0, firm j has already adopted a new technology". For $i \in \mathcal{O}(t_0)$, we define

$$\mathcal{M}_i^n(t_0) := \Big\{ M \in \mathcal{M}(t_0) \,\Big|\, M(i,1) = 1 \Big\} \quad \text{and} \quad \mathcal{M}_i^o(t_0) := \Big\{ M \in \mathcal{M}(t_0) \,\Big|\, M(i,1) = 0 \Big\}.$$

$\mathcal{M}_i^n(t_0)$ represents all the possible ways in which the technological vector could evolve, contingent on firm i adopting a new technology at the start of period t_0, and $\mathcal{M}_i^n(t_0)$ it the analogous for the case of no immediate adoption[9]. For any $j \in \mathcal{O}(t_0)$, any $M \in \mathcal{M}(t_0)$ such that $M(j, T+1-t_0) = 1$ represents a scenario where firm j never adopts a new technology within the phase.

[9]In terms of cardinality, $\#\mathcal{M}_i^n(t_0) = (T-t_0)^{\#\mathcal{O}(t_0)-1}$. This is not the case for $\mathcal{M}_i^o(t_0)$. The cardinality of this set is $(T-t_0)^{\#\mathcal{O}(t_0)} - (T-t_0)^{\#\mathcal{O}(t_0)-1}$.

Let $M \in \mathcal{M}_i^n(t_0)$, in order to compute firm i's payoff, should this matrix represent the way in which adoptions will take place, we construct a sequence of technology vectors $\{h_M(t)\}_{t=t_0}^T$ by defining their entries in the following way:

$$h_M(t_0)_j = \begin{cases} n, & \text{if } M(j,1) = 1, \\ o, & \text{if } M(j,1) = 0. \end{cases}$$

For t ranging from $t_0 + 1$ to T we set

$$h_M(t)_j = \begin{cases} n, & \text{if } M(j, t - t_0 + 1) = 1, \text{ or } h_M(t-1)_j = n \\ o, & \text{if } M(j, t - t_0 + 1) = 0, \text{ and } h_M(t-1)_j = o. \end{cases}$$

Notice that if $M \in \mathcal{M}_i^n(t_0)$, then $h_M(t)_i = 1$ for all $t \geq t_0$. Next we aggregate the payoffs corresponding to the $h_M(t)$'s:

$$\mathcal{P}_i^M = \sum_{j=0}^{T-t_0} (1+r)^t \phi_i(h_M(t_0+j), t_0+j) - (1+r)^{T-t_0} C_i. \tag{3}$$

The first term of this sum represents the per–period payoff stream associated to the path matrix M; whereas $(1+r)^{T-t_0}C_i$ is the discounted cost of adoption. When $M \in \mathcal{M}_i^o(t_0)$ the situation is similar, except for the possible adoption of a new technology at some future period. This is the time $t = \tau_i$, which was defined previously and corresponds to $t_0 - 1 + \min\{j \mid M(i,j) = 1\}$. Whenever $\tau_i \leq T$, the payoff associated to the corresponding $M \in \mathcal{M}_i^o(t_0)$ is:

$$\mathcal{P}_i^M = \sum_{t=t_0}^{T-t_0} (1+r)^t \phi_i(h_M(t_0+j), t_0+j) - (1+r)^{T-\tau_i} C_i. \tag{4}$$

If $\tau_i = T + 1$, then the term $(1+r)^{T-\tau_i}C_i$ disappears from Expression (4). The only difference between expressions (3) and (4) is the fact that in the latter $\tau > t_0$ and, accordingly, the (discounted) adoption cost kicks in after $t = t_0$.

For $k \in \{o, n\}$, we define the *payoffs vector* associated to $\mathcal{M}_i^k(t_0)$ as the vector with entries $\mathcal{P}_i^M$ ($M \in \mathcal{M}_i^k(t_0)$) ordered in increasing fashion, and we denote the latter by $\mathcal{V}_i^k(t_0)$. Firm i then assigns the following rating to the technological option k :

$$\Upsilon_i(t_0, k) := \left(1/\#\mathcal{M}_i^k(t_0)\right) \sum_{j=1}^{\#\mathcal{M}_i^k(t_0)} \mathcal{V}_i^k(t_0)_j.$$

If $\Upsilon_i(t_0, n) \geq \Upsilon_i(t_0, o)$, then firm i adopts a low–emitting technology at time t_0, otherwise it waits.

Remark 3.5 *$\Upsilon_i(t_0,k)$ represents firm i's expected utility under the assumption that all states of the world (represented by the elements of $\mathcal{M}_i^k(t_0)$) are equiprobable.*

We postpone the numerical examination of the long–term incentive to adopt a new technology under no–anticipation of technology adoption until Section 5. The numerical exercise compares the evolution of the path matrices – timing and level of technology adoption – within the framework presented above, and in the presence of a contingent price support instrument (the *cash–for–permits*) that we introduce below.

4 Trading strategy and new technology adoption with a price support contract

As investments in new technologies take place, the costs of pollution control fall. Under no–anticipation of new technology adoption, the allocation schedule remains unchanged, depreciating the permits value.

The system deviates from what the regulatory agency initially deemed optimal. It can be shown that, by adjusting the total number of permits, optimality can be reached *ex-post*. However, an amendment of a policy level is a rather exceptional event. When policy levels remain constant for long periods of time, an alternative to foster the long–term incentives to adopt new technologies is for the regulator to stand ready to buy back permits as needed, at an announced price. Arbitrage would ensure that the exchange value of allowances would stay above the announced price. Among others, this contract has been suggested by Roberts and Spence (1976), Laffont and Tirole (1996), and Biglaiser et al. (1995). We consider a similar mechanism and introduce a free–of–charge contract that is written on the final holdings of permits and is contingent on the technology status. We call this contract *cash–for–permits* (C4P for shorthand). We assume that adoption of new technologies is perfectly verifiable by the regulator, thus ruling out moral hazard.All firms that have adopted a new technology have access to C4P. In fact, their investment entitles these firms to as many C4P (put options) as allowances they hold. These options, however, are non–transferable, and they can only be exercised within the regulated period they are issued. When firms decide to exercise their options, the regulator buys back the permits at the pre–announced price, P_g.[10] One may think of the C4P (strike) price P_g as a minimum price guarantee contingent on the technology status. Below we show that the C4Ps create a (floating) price floor for the exchanged permits.

4.1 The impact of the C4P on the aggregate supply and the permit price

The introduction of the C4P has an impact on the number of the permits that firms in permit excess submit to the exchange and, therefore, affects the allowance exchange value. We maintain the notation $x_i(t+1,h)$, $\mathcal{D}(t+1,h)$ and $\mathcal{S}(t+1,h)$ used in the previous sections. If firm i has adopted a low pollution–emitting technology and it is in permit excess, then the quantity $x_i(t+1,h)$ can be divided into $x_i^e(t+1,h)$ and $x_i^c(t+1,h)$. The former indicates the number of permits submitted to the exchange, and the latter those that are "cashed–for–permits".

In parallel to Section 3.1, we must now study the generation of allowance prices considering how firms in permit excess balance their positions in C4P and market–exchanged permits. Assume that firm i is in permit excess and that it operates under new technology. Given that the other firms in excess have submitted $\mathcal{S}_{-i}^e = \sum_{j\neq i} x_j^e$ permits into the exchange market, firm i's choice of x_i^e yields a profit equal to:

$$\Psi_i^4\big(x_i^e, x_{-i}^e\big) := P_g(x_i - x_i^e) + x_i^e P \cdot \eta_{\mathcal{R}}\Big(-\frac{\mathcal{S}_{-i}^e + x_i^e}{\mathcal{D}}\Big). \tag{5}$$

Notice that the mapping $x \mapsto P_g(x_i - x)$ has constant slope $-P_g$. A relatively high P_g could then result in $x_i^e \equiv 0$ being the optimal exchange strategy for all firms that are in permit excess, and which operate under the new technology. Obviously, the condition $P_g < P$ should hold. This condition is sufficient to guarantee that markets will not shut down: it will not be optimal for all the firms to submit zero–supply schedules and exercise their C4P. The latter claim follows from the fact that

$$\frac{d}{dx}\,\Psi_i^4\big(x,0\big)\Big|_{\{x=0\}} = P - P_g.$$

If all other firms were to exercise their C4P, the marginal utility of firm i at zero would be increasing in its submissions to the exchange, hence it would find it suboptimal to abstain from trading permits.

For the ease of exposure, let us now split the sets $s(t+1,h)$ into $s^o(t+1,h)$ and $s^n(t+1,h)$. These sets represent, respectively, the firms that are in permit excess at the end of the period $[t, t+1]$ and that operate under the old technology throughout the period, and those that are in permit excess, but which have already adopted a new technology. Firms that belong to $s^o(t+1,h)$ simply operate as before; however, those in $s^n(t+1,h)$ will not submit permits into the exchange unless they can make at least P_g per unit

[10] It should be noted that under this contract the outstanding number of permits is modified via actions of the firms, and not due to direct intervention of the regulator.

of allowances traded. Again, we assume without loss of generality that total demand equals one, so that the payoff of a firm in $s^n(t+1,h)$, which submits x_i^e to the exchange, given that the other firms in permit excess have submitted K_1 is:

$$\Psi_i^4\big(x_i^e, x_{-i}^e\big) = P_g(x_i - x_i^e) + x_i^e P \cdot \exp\left\{\frac{(K_1 + x_i^e)^2}{(K_1 + x_i^e)^2 - 1}\right\}.$$

Similarly to Lemma 3.1, we have the following

Lemma 4.1 *For $i \in s^n(t+1,h)$, and any supply vector x_e^{-i}, the mapping $x \mapsto \Psi_i^4\big(x, x_{-i}^e\big)$ is maximized at a single point of $(0, x_i]$.*

With Lemma 4.1 at hand, the analysis of the existence of equilibria of the game $\mathcal{G} = \big\{[0, x_i(h)],\, \Psi_i^4\big\}_{i\in s}$ is analogous to that of Section 3.1, and the corresponding equilibrium will be denoted by $\{x_i^4\}_{i\in s}$. We may then conclude the existence of a (unique) equilibrium price $(\Pi^4)^*(t+1,h)$, which in turn keeps our description of the mechanics of trading mostly unchanged. The notable difference being that a firm in permit excess, operating under the new technology, has a profit:

$$\phi_i^4(t+1,h) = \,(\Pi^4)^*(t+1,h,)\cdot|x_i^4(t+1,h)| + P_g|x_i(t+1,h) -\, x_i^4(t+1,h)| + R_i.$$

Furthermore, as exemplified later in Figure 3, we have the following

Proposition 4.2 *Under identical primitives and identical triples $\big(T, \{N_i(t)\}_{i\in\mathcal{I}}, P\big)$, the following holds for all $t \in [0,T]$:*

$$(\Pi^4)^*(t+1,h) \geq \Pi^*(t+1,h).$$

Remark 4.3 *By construction, if $i \in s(t+1,h)$, for any h we have that*

$$\phi_i^4(t+1,h) \geq \phi_i(t+1,h).$$

The equality corresponds to the cases of boundary solutions or firms in permit excess that operate under the old technologies. From Proposition 4.2, we also get that if $i \in d(t+1,h)$ then

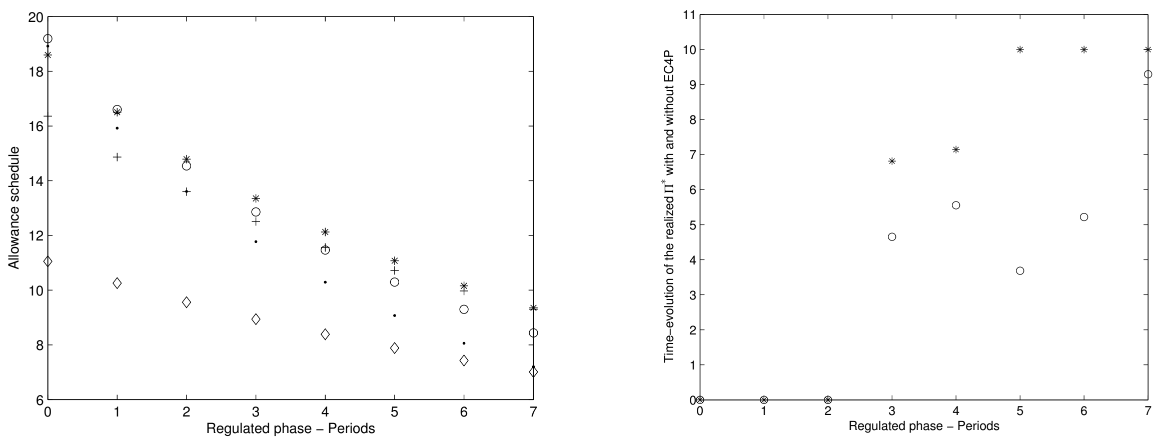
$$\phi_i^4(t+1,h) \leq \phi^i(t+1,h).$$

We have seen that the Nash–games that determine the allowance prices and govern the evolution of the technological vector are still well defined in the C4P–setting. It should be kept in mind that the payoff functions Ψ_4^i depend on the technology vector h. Furthermore, Proposition 4.2 indicates that it is in the interest of those firms operating under the new technology to reduce permits availability and increase their scarcity rents. Such a strategy's impact on prices increases the compliance and non–compliance costs of the firms in permit shortage. This in turn affects the evolution of the technological vector.

5 Numerical analysis of the dynamic technology adoption

In the previous sections we describe the firms' trading and technology adoption strategies without and with price support contact. We analytically show that, under identical primitives and identical triples $\big(T, \{N_i(t)\}_{i\in\mathcal{I}}, P\big)$, the allowance exchange value is higher under price support contact. The comparison of the evolution of the technological vectors $h(t)$ and $h^4(t)$, however, it is not straightforward. The caveat is that, depending on the exogenous economic shocks, firms may switch back and forth from being on the supply–side to being on the demand–side of the market. This results in the following phenomenon: over some periods the introduction of C4Ps benefit a certain firm, whereas she might be worse off over other periods (in comparison to its position had the C4Ps not been introduced). Thus, in this section we look more into the timing and level of technology adoption within a numerical framework, both in the original policy setup and with a price support contract.

We consider a five–firm, eight–period scenario. The penalty P has been set to 10 and $P_g = 5$. Later we analyze the effect of different levels of P_g. Figure 3(a) shows the allocation schedule and Figure 3(b) shows a possible evolution of the price process where $u_o \in [1.13, 1.15]$, $d_o \in [1.05, 1.07]$, $u_n \in [1.08, 1.10]$, and $d_n \in [1.02, 1.04]$.[11] The initial emission level is the same for each firm, $Q^i(0) = 100$, the time–constant probability is $q = 0.5$, and the cost vector to adopt the new technology is $C_n \in [100, 80]$. Having chosen these parameters, the first (last) firm is characterized by higher (lower) emissions and higher (lower) costs for technology adoption.



(a) Firm–wise allocation of permits.

(b) A possible realization of the permit price.

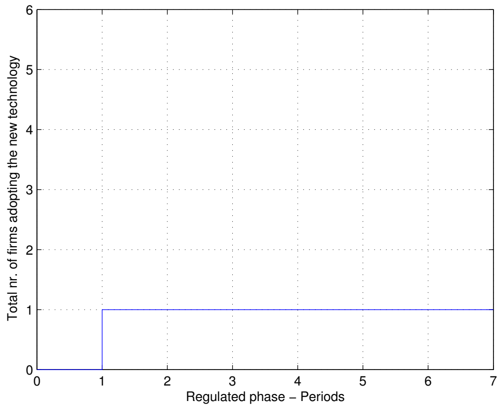
Figure 3: The left diagram represents the allocation path of permits to 5 regulated firms. The right diagram represents a possible path of the exchange value of allowances without C4P (points) and with C4P (stars).

As shown in Section 4.1, the allowance exchange value is higher in the presence of C4Ps (stars), Figure 3(b). Figures 4(a) and 4(b) show the evolution of the technology vector in aggregate terms: the adoption of the new technology is higher with C4Ps.

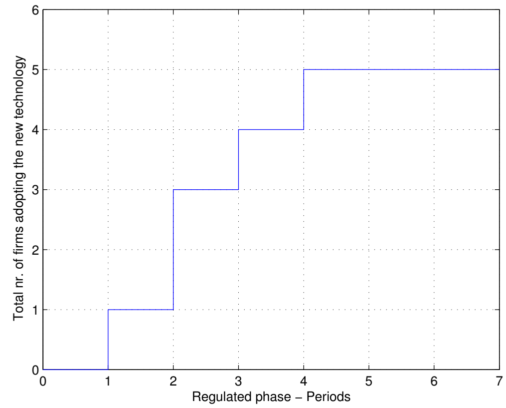
The rest of this section is dedicated to the examination of the impact of the minimum price guarantee on the timing and level of adoption of the new technology. We keep the same parameters as above, save for P_g, which varies between 1.5 and 4.5. The Figure below show that the higher the level of the policy, P_g, the higher the aggregate level of firms adopting the new technology. It is interesting to observe that by controlling the C4P policy level, P_g, the regulator is also able to affect the timing of the technology adoption. Figure 5(d) shows that by increasing price support, from $P_g = 3.5$ to $P_g = 4.5$, the regulator can accelerate the adoption of the new technology.[12]

[11] Recall that viewed from $t = 0$, the allowances price process is a random variable. Therefore, Figure 3(b) shows a possible path of the allowance price.

[12] A higher price guarantee increases the overall cost of the C4P. The regulator may want to balance the trade–off between inducing rapid technology adoption and having to pay high C4P costs. Such a decision may be part of the solution of the optimal policy levels. Alternatively, the problem could be analysed assessing the likelihood that the collection of compliance payments (permits auctions and penalty payments) renders the C4P disbursement neutral. In a working paper version of this paper we perform such an analysis.

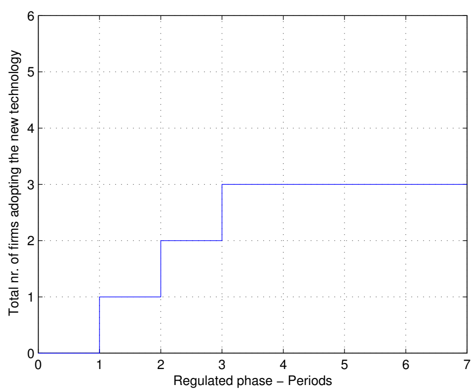


(a) Aggregate technology adoption without EC4P

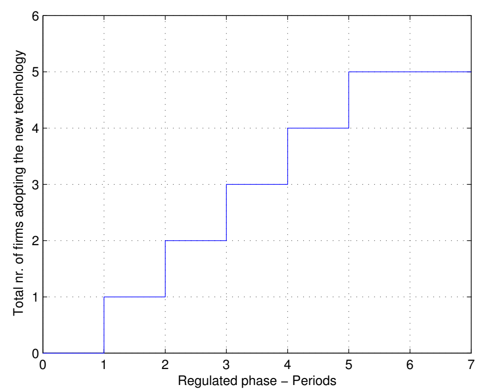


(b) Aggregate technology adoption with EC4P

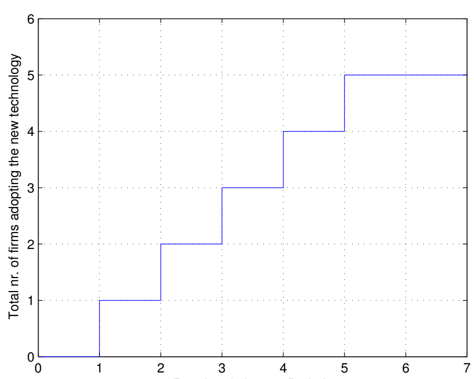
Figure 4: Evolution of the realized firm-specific technology vector in aggregate terms without EC4P (left diagram) and with EC4P (right diagram).



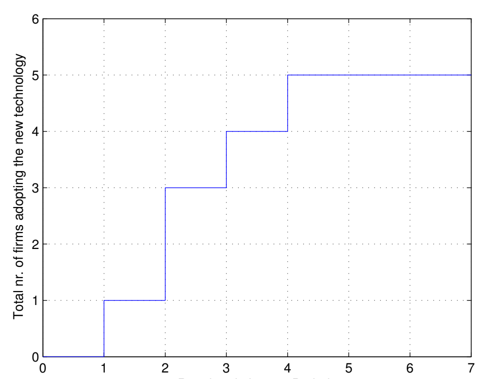
(a) Aggregate technology adoption for $P_g = 1.5$



(b) Aggregate technology adoption for $P_g = 2.5$



(c) Aggregate technology adoption for $P_g = 3.5$



(d) Aggregate technology adoption for $P_g = 4.5$

Figure 5: Evolution of the realized technology vector in aggregate terms with different levels of P_g.

6 Conclusions

Under a transferable permit scheme, the value of the allowances (scarcity rents) determines the incentives of regulated companies to invest in new, low pollution–emitting technologies. Permit schemes have traditionally been analyzed by focusing on the aggregate cost savings resulting from an industry–wide adoption of some new technology. In such a setting, most authors make the assumption that all firms adopt the new technology and then compare aggregate costs before and after adoption (Malueg (1989), Milliman and Prince (1989b), and Jung et al. (1996)). It has recently been shown that these aggregate cost savings differ substantially from a single firm's incentives to adopt a new technology (Unold and Requate (2001) and Requate (2005b)). These recent contributions highlight the fact that the possibility that a firm may free–ride on a decreasing permit price caused by other firms' investment in new technologies had previously been ignored. This firm may find it optimal not to adopt a new technology. Hence, both type of firms (those using old and new technology) coexist. As such, the number of firms adopting new technologies should be determined endogenously. By assuming perfect competition in the permit market, Kennedy and Laplante (1999) and Requate and Unold (2003) endogenize the number of firms that, in equilibrium, invest in new technologies. Here the price of permits depends on the number of firms that have adopted the new technology. The firms, however, fail to internalize the externalities they impose on others via their investment decisions an their corresponding impact on the market price of permits. We have relaxed such an assumption and investigated technology adoption behaviour under imperfect competition in the permit market. The inter–dependence between technology adoption and the permit price has been explicitly modelled in our framework: the number of firms that decide to adopt a new technology depends on their (rational) expectations of the future permit price; the future price of permits hinges on the number of firms that adopt a new technology. In particular, technology adoption is contingent on the future permits' scarcity and is ultimately reflected in the allowances price. Both aggregate demand and the aggregate supply are the driving forces of the permit price. The latter being a function of uncovered pollution emissions (permits demand), the level of unused emission permits (permits supply), and the current technological status. We have argued that under imperfect permit market competition, the aggregate permits supply does not necessarily equal to the number of unused permits, and based on the latter we have modelled the aggregate permit supply as the (unique, pure–strategies) equilibrium of a non–cooperative Nash game.

Under no–anticipation of new technology adoption, the system might deviate, as investments in new technology take place, from what the regulatory agency initially deemed optimal. By adjusting the total number of permits, optimality could be recovered ex–post. Policy levels, however, tend to remain constant for long periods of time. This is a typical scenario in real politics. Following the suggestions by Roberts and Spence (1976), Laffont and Tirole (1996), and Biglaiser et al. (1995), we have introduced a price–support contract: The Cash–for–permits (C4P). This instrument provides an alternative to permits adjustment and may be used to foster the firms' long–term incentives to adopt new technologies. We have modified the Nash–game that served to set the expected allowance prices and governed the evolution of the technological vector in the no–anticipative setting, which yielded a well–defined price–generation methodology. This has allowed us to show, theoretically and numerically, that the price support contact creates a price floor that can be interpreted as a (floating) minimum price guarantee. We have numerically examined the impact of the minimum price guarantee on prices, as well as on the number of firms that adopt low pollution–emitting technologies. The timing of such adoptions was also part of our analysis. We have found that the higher the minimum price guarantee, the higher the aggregate level of firms adoption and, quite interestingly, the earlier the adoption of the new technology.

We have modelled the investment technology in a somewhat rigid fashion: a one–shot, irreversible choice. A very interesting, yet technically challenging option to be studied would be to allow for a continuous choice in (irreversible) investment. This would result in a continuum of Nash games, both along the equilibrium path and off it, as the phase evolves. Secondly, investments create purely private benefits.

A Appendix

Proof of Lemma 3.1 We assume that $\mathcal{D} > 0$, otherwise there is no demand for allowances, hence no market, and the maximizer is trivial. We must show that the mapping

$$x \mapsto x \exp\left\{ \Big(\frac{K_1 + x}{K_2}\Big)^2 \Big/ \Big(\Big(\frac{K_1 + x}{K_2}\Big)^2 - b \Big) \right\},$$

where $b = \left(\frac{K_1+x^i}{K_2}\right)^2$, $K_1 = \mathcal{S}_e^{-i}$ and $K_2 = \mathcal{D}$ is maximized at a single point of $[0, x^i]$. By rescaling if necessary, we may assume without loss of generality that $K_2 = 1$. Moreover, we may assume that $K_1 + x_i \leq 1$, given that under the previous assumption $\eta_{\mathcal{R}} \equiv 0$ for any value larger than 1. Initially we assume that $K_1 + x_i = 1$, i.e. firm i has the ability, given K_1, to fully satisfy the demand for allowances. Since in such a case the values of the mapping under investigation are strictly positive on $(0, x^i)$, we need only to seek interior maximizers. The first order conditions yield the equation

$$L(x) := (K_1 + x)^4 - 4(K_1 + x)^2 + 2K_1(K_1 + x) + 1 = 0.$$

We have that $L(0) = (K_1 - 1)^2 > 0$, and $L(x^i) = -2 + 2K_1 < 0$. It follows from the Intermediate Value Theorem that L has a root x_{i_0} in $(0, x^i)$. To show uniqueness, we note that L'' changes sign only once on $[0, \infty)$, which given the general shape of the graph of a fourth–degree polynomial implies there are only two roots in this interval. Since $L(x^i) < 0$ and $\lim_{t\to\infty} L(t) = \infty$, we conclude that the remaining root lies beyond $x = x_i$. If it were the case that $K_1 + x_i < 1$, then either $x_{i_0} \leq K_1 + x_i$, in which case the previous result holds, or the maximizer is precisely x_i. $\square$

Proof of Lemma 3.2 For $x \in [0, x^i]$, the mapping

$$K_1 \mapsto x \exp\left\{ \Big(\frac{K_1 + x}{K_2}\Big)^2 \Big/ \Big(\Big(\frac{K_1 + x}{K_2}\Big)^2 - b \Big) \right\}$$

is continuous. Notice that x_{-i_j} is a relevant statistic for $\Phi_i(x_{i_j}, x_{-i_j})$ only through $\sum_{k\neq j} x_{i_k}$, and clearly the mapping

$$(x_{i_1}, \ldots, x_{i_{m_s}}) \mapsto \bigotimes \sum_{k\neq j} x_{i_k}$$

is continuous. It follows immediately that the mapping $(x_{i_j}, x_{-i_j}) \mapsto \Phi_i(x_{i_j}, x_{-i_j})$ is continuous over $\bigotimes[0, x_{i_j}]$, which finalizes the proof. $\square$

Proof of Lemma 3.4 The first point follows from the fact that the best–response path of a firm whose supply–constraint is binding reaches and is absorbed by the corresponding $x^i(h)$. Next we assume $\#s = 2$ for notational simplicity. Since for a fixed x_1, the expression

$$1 - \frac{2x_1(x_1 + x_2)}{((x_1 + x_2)^2 - 1)^2},$$

which corresponds to the first order conditions of firm 1, is decreasing in x_1, if $x_{1_h} < x_1^*(t+1, h)$, then $\tilde{x}_2(t+1, h) > x_2^*(t+1, h)$. In what follows we drop the arguments $(t+1, h)$ for clarity. The question remains whether or not the increased supply by firm 2 over the unconstrained–equilibrium level fully compensates the decreased supply of firm 1, as to leave aggregate supply unchanged. The answer is no. If firm 2 were to offer $x_1^* - \tilde{x}_1 + x_2^*$, we would have

$$1 - \frac{2\big(x_1^* - \tilde{x}_1 + x_2^*\big)\big(x_1^* + x_2^*\big)}{\big(\big(x_1^* + x_2^*\big)^2 - 1\big)^2} = 1 - \frac{2\,x_2^*\big(x_1^* + x_2^*\big)}{\big(\big(x_1^* + x_2^*\big)^2 - 1\big)^2} - \frac{\big(x_1^* - \tilde{x}_1\big)\big(x_1^* + x_2^*\big)}{\big(\big(x_1^* + x_2^*\big)^2 - 1\big)^2} < 0.$$

The inequality follows from the fact that the first two terms on its left hand side add up to zero (the first order condition for the unconstrained equilibrium) and $x_1^* - \tilde{x}_1 > 0$. We conclude that $\tilde{x}_1 + \tilde{x}_2 < x_1^* + x_2^*$, which in turn implies $\tilde{\Pi}(t+1,h) > \Pi^*(t+1,h)$. □

Proof of Lemma 4.1 As in the proof of Lemma 3.1, we first assume that $K_1 + x_i = 1$. Let

$$f(x) := P_g(x_i - x) + xP \cdot \exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\}$$

and

$$g(x) := f'(0)x + P_g x_i.$$

The graph of the function g in simply the tangent to the graph of f at $x = 0$. We observe that for $x \in (0, x_i)$,

$$\begin{aligned}
g(x) - f(x) &= (f'(0) + P_g)x - xP \cdot \exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\} \\
&= xP \cdot \exp\left\{\frac{(K_1)^2}{(K_1)^2-1}\right\} - xP \cdot \exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\} \\
&> 0.
\end{aligned}$$

In other words, the graph of f is strictly under the graph of its tangent at $x = 0$. We also have that $f'(0) = -P_g + P \cdot \exp\left\{\frac{(K_1)^2}{(K_1)^2-1}\right\}$. If this quantity were to be non–positive, then f would be maximized at $x = 0$. On the other hand, if $f'(0) > 0$ and $f'(x_i) < 0$, then there is $x_{i_0} \in (0, x_i)$ such that $f'(x_{i_0}) = 0$. Moreover, $x \mapsto xP \cdot \exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\}$ is a quasiconcave (thus single–cusped) mapping, hence so is $x \mapsto f(x)$. We may then conclude that x_{i_0} is unique. The case where $K_1 + x_i < 1$ follows in a similar fashion, but $f(x_i) > 0$. Nevertheless, f remains quasiconcave, hence maximized at a single point.

□

Proof of Proposition 4.2 It suffices to show that under any circumstance, the best response of a firm that is in permit excess is lower or equal (in terms of units or permits submitted to the exchange) with C4Ps than it would be without C4Ps. Trivially firms that find themselves in permit excess, but which have not change technology, will have the same best responses as before to a given submission level K_1. We assume that the best responses of firm i to K_1 are interior (i.e. they belong to $(0, x_i)$), since otherwise we find boundary solutions where x_i is submitted. Below we write the first order conditions for the interior solutions. The case of no C4P corresponds to the solution of the equation

$$\exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\}\left(1 - \frac{2x(K_1+x)}{((K_1+x)^2-1)^2}\right) = 0, \tag{6}$$

whereas in the presence of EC4P we must find the root of

$$\exp\left\{\frac{(K_1+x)^2}{(K_1+x)^2-1}\right\}\left(1 - \frac{2x(K_1+x)}{((K_1+x)^2-1)^2}\right) = \frac{P_g}{P}. \tag{7}$$

Since the mapping $x \mapsto -\frac{2x(K_1+x)}{(K_1+x)^2-1)^2}$ is decreasing, the root of Equation (7) on $(0, x^i)$ is smaller than that of Equation (6). By virtue of Lemma 3.4, we know that any additional permits submitted by firms who are not eligible to cash–4–permits will not restore the total supply to its pre–C4P levels, which concludes the proof.

□

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